

2019 Enrolment The 1st
Japan University Examination
Advanced Mathematics

Examination Date: November 2017

(90 min)

Do not open the examination booklet until the starting signal for the exam is given.

Please read the following instructions carefully.

Please fill in the examinee no. and name below.

Instructions

1. The booklet contains 3 pages.
2. The answer sheet is one piece of one sided paper.
3. In the case that you notice there are parts in the booklet where the print is not clear or there are missing pages or misplaced pages, or the answer sheet is soiled, raise your hand to report to the invigilator.
4. There are 3 questions to be answered.
5. Fill the examinee no. and name in the answer sheet.
6. Use black pencil to write answers in the designated section in the answer sheet.
7. Memos and calculations can be written on the examination booklet.

Examinee'sNo.	Name

1

Consider the circle C and straight line l on xy plane.

$$\text{Circle } C: x^2 + y^2 - 6x - 2y + 5 = 0$$

$$\text{Line } l: (3a + 1)x - (3a + 5)y - 6a + 22 = 0 \quad (a \text{ is a real constant})$$

- (1) Calculate the coordinate of the centre and the radius of circle C .
- (2) It is known that line l must go through a fixed point whatever the value of a is. Calculate the coordinate of the fixed point.
- (3) Calculate all values of a , in which line l is tangent to circle C .
- (4) Assume that the value of a is the largest value calculated in question (3).

Here, assume that Circle C and circle C_1 are symmetrical about line l , and circle C_2 is circumscribed to circle C_1 and touches the x -axis.

- (i) Calculate the coordinate of the center of circle C_1 .
- (ii) Find an equation of figure that the center of circle C_2 is constituting.

2

Consider that a is a positive constant, and $a \neq 1$.

(1) Here function of x is defined as :

$$y = 4^x - a \cdot 2^{x+1} + 3a^2 + a$$

and assume $2^x = t$

- (i) Calculate the range of values of t .
- (ii) Express y in terms of t and a .
- (iii) Express the minimum value of y in term of a .

(2) Here, function of x is defined as :

$$f(x) = \log_a(4^x - a \cdot 2^{x+1} + 3a^2 + a)$$

- (i) If $a = 2$, calculate the minimum value of $f(x)$.
- (ii) If the minimum value of $f(x)$ is 3, calculate the value of a .
- (iii) If $f(x) \leq 0$ whatever the value of constant x is, then calculate the range of value of a .

3

Consider a and b are both positive integers, and p and q are both real constant.

The integral expression of x is as follows:

$$f(x) = x^3 - 3ax^2 + (2a^2 + 2a - b - 2)x - a^2 + 2a + b + 3$$

When this equation is divided by $x^2 - 2ax - 2$, it gives $x - a$, and the remainder is $px + q$.

- (1) Express each p and q in terms of a and b .
- (2) Set that $\beta = a + \sqrt{3}$, and the integer part of β is r .
 - (i) Express r in terms of a .
 - (ii) If $f(r) = 0$, then calculate all the combinations of the values of (a, b) .
 - (iii) Set that $f(\beta) = \sqrt{3} + c$ (c is a rational number), then calculate the value of a, b, c .